Exponential Growth Theory of Mechanical Investing:
The XG and RRS Screens, and the XG_RRS Spreadsheet

by Loren Cobb

(email: cobb@Aetheling.com)

Abstract

This paper discusses a simple idea for estimating the growth rates of total returns for stocks that appear to be growing on an exponential trajectory. It is intended for use in conjunction with the stock screening methods of Mechanical Investing. A spreadsheet written by William Lipp for constructing and backtesting these screens is introduced, with examples of its use.

Introduction

Stocks appreciate in value in many ways. Some are cyclical, others are very sensitive to the overall market, and yet others grow only sporadically, with sudden lurches upward or downward whenever relevant news items hit the wires. But to my mind, the perfect style of growth for Mechanical Investing is ***exponential***. Stocks that appreciate in this manner have a constant total return over time. The graph of the stock price of such a stock is a beautifully exponential curve, whose rate of increase is always proportional to its current price. If a stock were perfectly exponential, then we would all make a fortune investing in this stock and its options, with no risk at all. In the real world, which I occasionally visit, things are more chancy. No real-world stock has a perfectly exponential trajectory, but some come close for periods measured in months and sometimes years.

For example, Figure 1 below shows the stock price of JDSU plotted on a logarithmic scale. If the trajectory is exponential then the graph should look like a straight line. Indeed, there was a time between November 1998 and November 1999 when the graph was remarkably linear. This is when people made a lot of money investing in JDSU and its options. It's intriguing to observe that now, in June of 2000, JDSU has returned to its projected line of exponential growth, after having been influenced by the infamous "internet bubble."

Figure 1: The split-adjusted stock price of JDSU on a logarithmic scale, over the last two years.

Now let's focus on the fluctuations. The day-to-day *increases* in log(Price) for a stock make a series of numbers that is of interest because it is nicely behaved, in the sense that the data are close to normally distributed. The mean of this series of numbers is the average daily gain in log(Price), while the standard deviation is a measure called the historical volatility of the stock. These two statistics, μ and σ respectively, are used to make the projections on which the Exponential Growth screens are based.

Since μ is the mean daily change in log(Price), the expected daily return is $\exp[\mu]$. Over t days it will be $\exp[\mu t]$. To convert this to a growth percentage, subtract one and multiply by 100. But this is only the expected gain. What about its variation? As a rule of thumb, two-thirds of the time the daily gain will be in the range between $\exp[\mu - \sigma]$ and $\exp[\mu + \sigma]$. This is why we calculated σ. The range $\exp[\mu \pm \sigma]$ is sometimes called the *one-sigma range*. (For those unfamiliar with the notation, $\exp[x]$ is just another way of writing e^x).

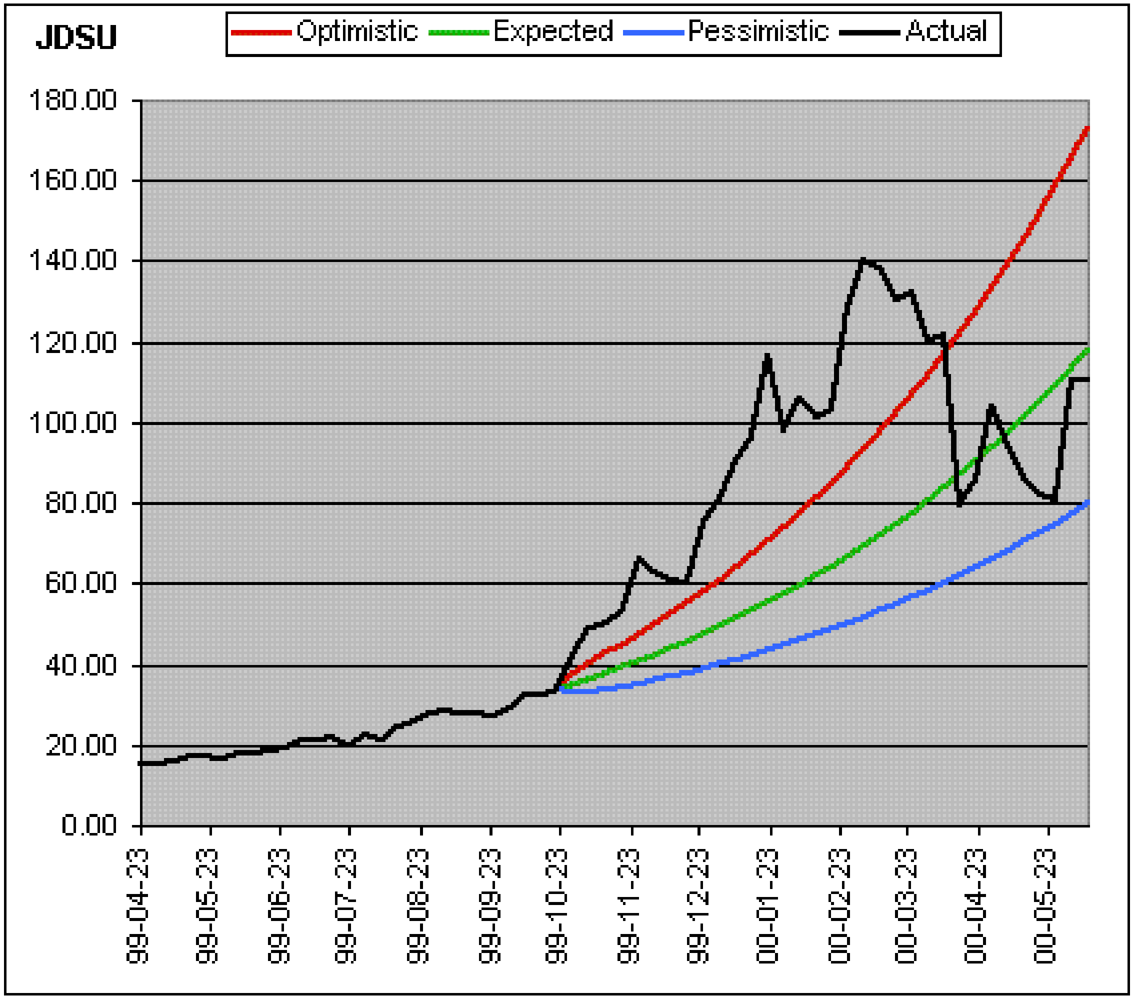

Figure 2: Expected growth (green) and the one-sigma bands (red and blue) for JDSU, from 23 April 1999. These projections are based on six months of prior data. Unlike the previous figure, which was shown on a logarithmic scale, the stock price of JDSU (split-adjusted) is graphed here on a rectilinear scale.

To project the range further into the future, we need one additional fact about random time series. When we add two independent random increments together, as we must to project two days ahead, the standard deviation of the sum is **not** 2σ. Instead, it is $\sigma\sqrt{2}$. In general, when we add t independent random increments, for example to project t days into the future, then the standard deviation of the sum of the t increments is $\sigma\sqrt{t}$. Therefore, the one-sigma range for the return of our stock after t days will be between $\exp[\mu t - \sigma\sqrt{t}]$ and $\exp[\mu t + \sigma\sqrt{t}]$. Again, to convert these into growth percentages, subtract one and multiply by 100.

The fundamental model for stock price change that we are using is the stochastic differential equation $d(\log X) = \mu dt + \sigma dw$. From this SDE one can derive that stock price X at time t is lognormally distributed, or in other words, that $\log(X_t)$ is normally distributed with mean μt and variance $\sigma^2 t$.

Anyone who wishes to see the mathematics behind these ideas in greater depth and detail should consult **Options, Futures, and other Derivative Securities, 2nd Edition**, by John C. Hull (Prentice-Hall, 1993), pp. 210-217.

One could reasonably call the limits of the one-sigma range "pessimistic" and "optimistic" projections. As a rule of thumb, if the stock continues to grow the way it has in the past, and if deviations from the exponential growth curve are statistically independent, then the actual rate of return will be within its one-sigma range about two-thirds of the time. Pessimists will most likely be comfortable focusing on the lower end of the one-sigma range, while optimists will happily study the higher end. The bottom of the one-sigma range for growth rates has often been called the *"risk-adjusted growth rate."* Investors who focus on the *expected* rate of return can be characterized as unconcerned with the size of the random fluctuations that may affect their stock choices. In contrast, pessimists worry that the fluctuations will go against them, while optimists tend to assume that they will break in their favor.

Figure 2, below, shows how these projections look in the case of JDSU, projecting forward from 23 April 1999, based on six months of prior daily data.

I present the JDSU case because it demonstrates several important points: first, even one year of almost perfect exponential growth with low volatility does not guarantee that the low volatility will continue into the future. An investor who purchased JDSU on 23 October 1999, as many did, would have experienced first greater-than-expected growth over the next five months, then a sickening plunge, then a sudden recovery — in other words, much greater than expected volatility.

Second, investor experience with JDSU during the rest of calendar year 2000 illustrates a most important point: every exponential trend comes to end sooner or later. The stock price of JDSU fell dramatically during the bear market of 2000-2001, much to the chagrin of those late investors who finally purchased the stock in February, hoping to catch the same wave that so many others had ridden for so long.

The JDSU case illustrates a third and more subtle phenomenon as well: serially correlated disturbances. Once JDSU had broken the pattern and diverged from its pure exponential trajectory, it engaged in quite a substantial excursion. In plain English: *large external disturbances tend to endure longer than one might expect, sometimes much longer.* The formulas used here to project the one-sigma range do **not** take into account the possibility of serially correlated disturbances; instead they assume that all disturbances are independent. The appropriate statistical correction is not difficult, but it lies outside the scope of this paper. Meanwhile, it would be fair to assume that the calculated one-sigma ranges are not quite as broad as they ought to be.

Screening Theory

Screening for maximum projected growth in total return for a stock based on an exponential growth model is really not appropriate for the majority of stocks, because most simply do not grow exponentially. Some are cyclical, some are too sensitive to the overall market, others grow only sporadically, with sudden news-induced lurches upward, and yet others grow simply because their market prices have become noticeably low compared to their intrinsic values. Many investors make very satisfactory profits while ignoring the small subset of stocks that are growing exponentially. The following screening theory is not intended to be understood as a general theory of investing. Instead it is much less ambitious, trying only to deduce from first principles how to locate those stocks whose growth is so regular and exponential as to provide substantial confidence that investing will be prove to be profitable.

Partly because there are so many different ways in which stocks grow, there is no universal “prediction formula” that can predict the growth rate of a stock based on its fundamental characteristics, its history of price changes as illustrated on a chart, and on the characteristics and behavior of the larger market. Instead, experience teaches that one must first focus on a well-defined segment of the market, and search within this segment for stocks likely to perform well. Even within a severely restricted and homogeneous segment, however, the search for a useful price prediction formula almost always fails. The likely reason for this failure, I believe, is that the proportion of stocks within the segment that actually grow substantially almost always turns out to be small compared to the total number of stocks in the segment, thus having too little leverage in the regression analysis to account for much.

The alternative is to search for stocks within the segment that occupy extreme positions with respect to one or more variables, in the hope that these extreme stocks will also exhibit unusual growth characteristics. For example, one could look within the segment of small capitalization stocks with positive recent growth and earnings for those particular stocks that have the greatest growth rates. This general technique is called “screening.”

Why does screening work, when highly sophisticated statistical prediction methods fail? I believe, though I cannot prove, that there are mathematical reasons why screening has been successful. Because they are rooted in geometry and topology, these reasons are easy to explain and comprehend through the use of pictures.

Suppose that we were supermen who could visualize space in any number of dimensions, not just the conventional three. Then we could “see” stock growth potential as a function of many different variables simultaneously. It would like look a surface, and our task as investors would be to find the highest point on the surface. Not being supermen, unfortunately, our ability to visualize this hypersurface of growth potential is sadly constrained. We can see only three-dimensional sections of the surface, one at a time, each looking something like Figure 3. In the middle of the figure, representing stocks of average characteristics, we expect to see average growth. If we are lucky in our choice of variables to examine, we may see possibilities for optimizing growth as we look to the extremes of one or more variables.

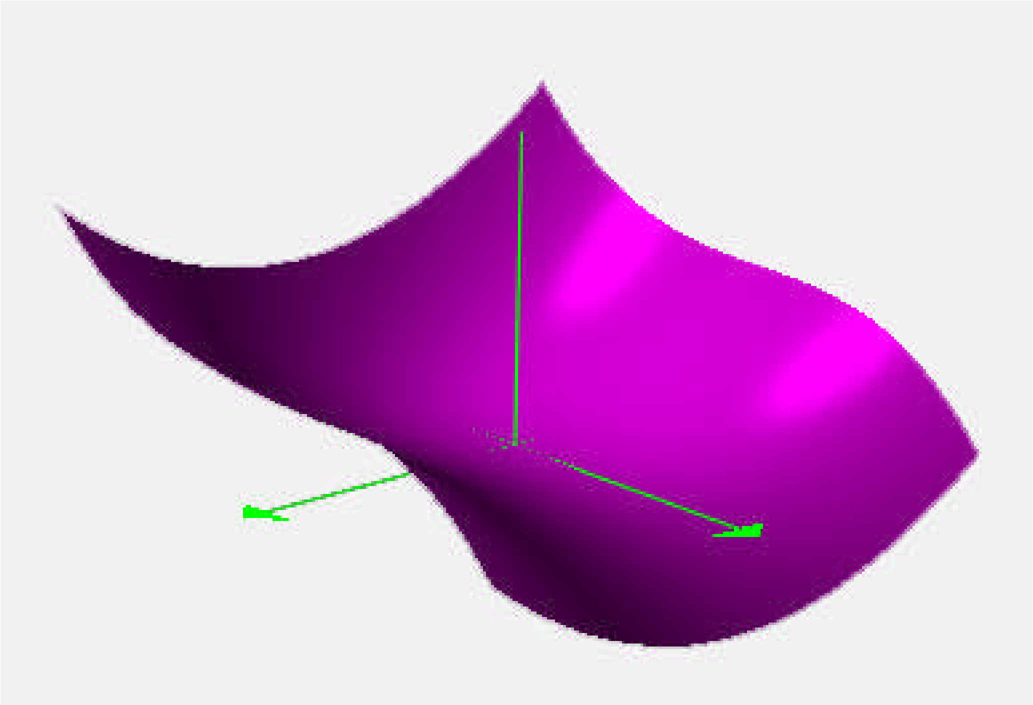

Figure 3: Optimizing growth in two dimensions.

There are two geometries that are simpler and more likely than any others, and they are illustrated in Figure 3, along the two horizontal dimensions depicted by green arrows. In either case, the optimum will lie somewhere along the boundary of the figure. Our task is to move from the middle out to the boundary in steps that will take us to somewhere near the optimum. Screening works because that is exactly what it does: by sorting our set of stocks according to a variable and then focusing on the extreme cases in the sort, we have moved our focus towards the boundary of the surface in Figure 3, along the dimension that corresponds to the variable on which we did the sort. In some cases there might be an optimum that is not located on the boundary — these are the cases where screening methods fail, and a completely different approach is required.

Screening for Exponential Growth

It is clearly not sufficient merely to screen for high growth rates, because we also need predictability. The fundamental idea behind the Exponential Growth family of screens is this: *by sorting stocks according to an index of growth that penalizes stocks with high volatility, we can select those high growth stocks that will grow more reliably*. If the two dimensions in Figure 3 are growth rate (backwards along the green arrow from lower right to upper left) and volatility (forwards along the arrow from upper right to lower left), then an index that combines the two is designed to move diagonally towards the peak in the rear.

There are in fact many ways that growth can be penalized for volatility. The simplest (and therefore always the natural starting point) is a simple subtraction process. This is another version of an old idea: risk-adjusted growth. Just as earlier we spoke of an “optimistic” and a “pessimistic” view of anticipated fluctuations in stock price, we can also rate the degree of risk-adjustment in our index by the amount of volatility that is added or subtracted from the growth rate. Here are the simple cases that have been studied thus far, with names that describe the risk tolerance of their users:

1. ***Risk-Averse:*** subtract twice the volatility.
2. ***Pessimist:*** subtract the volatility.
3. ***Risk-Neutral:*** do not adjust for volatility.
4. ***Optimist:*** add the volatility.

The Risk-Neutral case is equivalent to the standard “Relative Strength” screens of Mechanical Investing. The Optimist case is almost certainly for bull markets only, and may be most appropriate for those investors who love to gamble when their hands are hot and the gods are smiling. The rest of us will be more comfortable with some degree of cynical pessimism.

Here are specific details for the screening method:

1. Start with historical data, D trading days into the past, for a set of N stocks. Take the logarithm of the daily closing stock prices (with possible additions for dividends).

2. Compute the growth rate of each stock, using either mean daily increase in log price (the "XG" method), or the regression slope of log price as a function of time (the method developed and researched by BarryDTO under the name Regression Relative Strength, or "RRS").

3. Compute the historical volatility of each stock. Volatility is defined as the standard deviation of the daily increases in log price of the stock.

4. Choose a forward projection time in days, called P, and convert the mean and volatility to this time period (i.e. multiply the mean by P, and the standard deviation by the square root of P).

5. Compute your preferred risk-adjusted index of mean growth, by subtracting a suitable multiple of the volatility from the mean growth.

6. Sort all stocks by this index. Proceed, as with any other MI screen, to purchase equal dollar amounts of the top K stocks in the sort. Hold for a specified period of time, then rebalance.

Given the heavy requirements in this method for historical data retrieval and statistical analysis, an automated approach is an urgent necessity. William Lipp has written an Excel spreadsheet that can be used to carry out this chore, as described in the next section.

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Then the Calculate sheet will be populated, and the results copied here. Calculate can be ch

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				log(e) of Total Return			
Symbol	End Date	Trade Date	Closing Price	63 days	126 days	189 days	252 days
AAS	1-Feb-01	1-Feb-01	$ 47.500	0.29481	0.57536	1.28919	0.92954
ABFS	1-Feb-01	1-Feb-01	$ 20.563	0.86531	0.71162	0.61779	0.53856
AEOS	1-Feb-01	1-Feb-01	$ 37.250	1.91417	2.17719	1.66218	0.61310
ALSI	1-Feb-01	1-Feb-01	$ 29.813	0.37789	1.12343	0.96104	0.76484
AMCC	1-Feb-01	1-Feb-01	$ 70.125	-0.19482	-0.01244	0.29248	0.52534
APOL	1-Feb-01	1-Feb-01	$ 36.625	1.40271	0.77709	0.92875	0.83336
AW	1-Feb-01	1-Feb-01	$ 15.100	1.88001	0.92681	1.23058	0.95468
AZA	1-Feb-01	1-Feb-01	$ 40.780	0.05233	0.42716	0.82476	0.77035

Figure 4: A portion of the **Main** sheet.

William Lipp's XG_RRS Spreadsheet

The XG_RRS spreadsheet was designed and written by William Lipp to give mechanical investors an easy and flexible way to perform their own screening using the general principles and methods of the Exponential Growth theory. It may also be used for backtesting, but limitations in the Yahoo Historical Quote database will prevent backtesting beyond two or three years in the past. Figure 4, below, shows a small portion of the Main sheet in this spreadsheet.

Basic Instructions:

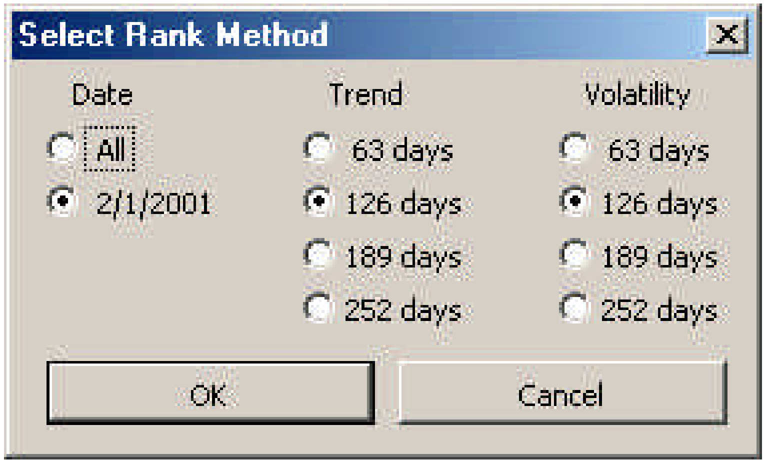
1. Because the XG_RRS spreadsheet downloads historical quotes from the web-based Yahoo Historical Quote facility, your computer must be connected to the internet.

2. In the first column of Main sheet, list all of the stock symbols in your screening universe.

3. In the second column, specify the last date for which historical quotes are to be obtained.

4. Click the **Recalculate** button, found in the upper left-hand corner of the Main sheet. As each vector of historical quotes is downloaded and analyzed, the corresponding row of the spreadsheet will be populated with statistical results. This will take several minutes for 100 stocks.

5. Move to the Rankings sheet, and click the **Rank** button. This dialog appears:



6. Click the **All** radio button only if there are multiple dates for the same stock listed in the Main sheet (an unusual but possible circumstance). Generally all stock histories will end on the same date, which will be specified in the second radio button.

7. Click the appropriate radio button for the number of days in the past over which to calculate the mean rate of growth (labeled “Trend”).

8. Click the appropriate radio button for the number of days in the past over which to calculate the historical volatility.

9. Click the **OK** button to generate the rankings, which will appear on this sheet. These rankings are generated by sorting on risk-adjusted projected annual growth.

The spreadsheet was designed to be easily modified for research purposes by wizards who are proficient in Excel. For example, alternative measures of historical volatility or the rate of growth can be introduced, without even modifying any of the Visual Basic programming — only some spreadsheet formulas would need to be changed. Similarly graphs and charts could be added, without too much difficulty.

Interpreting the Results

Exponential Growth screens use projections to make their rankings, and projections are troublesome to interpret correctly — the human mind always wants predictions instead. Projections are not predictions! To make a prediction from a projection, all of its assumptions must be given. Thus a defendable prediction statement based on a use of the XG_RRS spreadsheet might read as follows:

“If the price of ADCT continues to grow along the estimated exponential path that it followed during the six months prior to 9 June 2000, with the same level of volatility, and if nothing in the market or the internals of ADC Telecomm occurs which might change the growth dynamic, then the graph of the price of ADCT over the next year will fall within the one-sigma range (growth rates between 185% and 802%) about two-thirds of the time.”

The three huge caveats in this statement are the preliminary “if” conditions. Unfortunately, the chances are excellent that **none** of these conditions will be fulfilled, thus invalidating the entire prediction. It is an error to suppose that the probability statement at the end covers these contingencies. It does not — it only covers the normal fluctuations expected on the basis of the observed historical volatility. The probability statement applies only if all three preliminary conditions hold, and they may well not hold. Therefore, it is best not to try to make predictions out of these projections.

Using the Results

Usage is another matter. I am personally comfortable with basing some of my investment decisions on these projections, because I believe that the rankings generated by this method actually reflect the relative quality of these potential investments reasonably well, and arguably better than many of the other mechanical screens in the “Relative Strength” family.

Extensive backtesting of the RRS method has been performed by BarryDTO, and reported in posts to the Mechanical Investing board of the The Motley Fool. See, especially, these posts in his “Daily Data Analysis” series: 81373, 81414, 82733, 83661, 87544, 89970, 90073.

May all random fluctuations break in your favor!

Loren Cobb, Ph.D.
Carbondale, Colorado.